Lesson 15 - Solids of Revolution - Washers, part I

I. Washers vs. Disks

II. Examples

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## I. Washers vs. Disks

disks


$A = \pi R^2$

$V = \pi R^2 \Delta y$ → $dy$

washers


$A = \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$V = \pi (R^2 - r^2) \Delta y$

$dy$

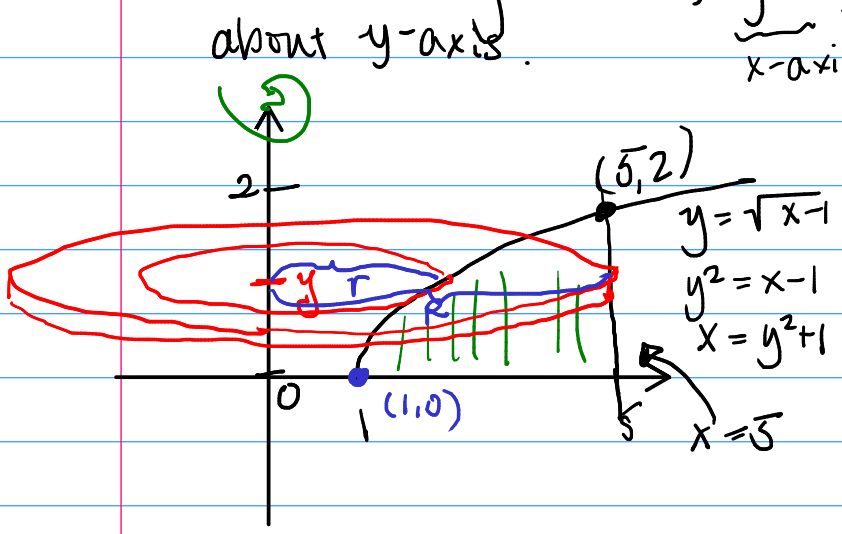
Reminder: Graphs help.

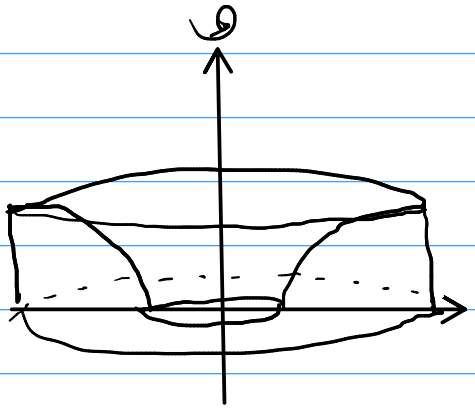
## II. Examples

**EX** Find the volume of the solid obtained by rotating the region enclosed by

$y = \sqrt{x-1}$, $y = 0$ (x-axis), $x = 5$

about y-axis.



intersection pts

$\begin{cases} y = \sqrt{x-1} \\ x = 5 \end{cases} \Rightarrow y = \sqrt{5-1} = \sqrt{4} = 2$

OR

$\begin{cases} x = y^2+1 \\ x = 5 \end{cases}$

$5 = y^2+1$

$y^2 = 4$

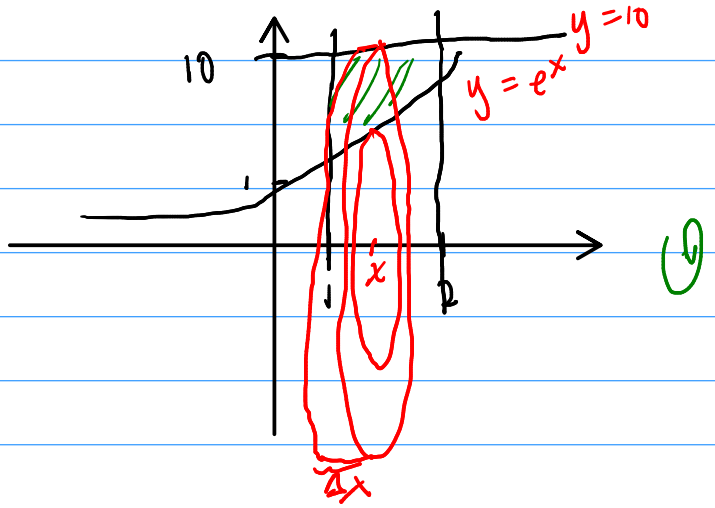
$y = 2$ OR ~~$-2$~~

$R = 5$ $\qquad r = y^2+1$

$$V = \int_0^2 \pi\left[5^2 - \underbrace{(y^2+1)^2}_{y^4+2y^2+1}\right] dy$$

$$= \int_0^2 \pi\left[24 - y^4 - 2y^2\right] dy \qquad \text{Power rule} \qquad \pi\left(\frac{544}{15}\right)$$

**EX** Find the volume of the solid obtained by revolving the region enclosed by $y = e^x$, $x = 1$, $x = 2$, and $y = 10$ about the x-axis.




washers
@ x-axis (horizontal)
$\Rightarrow$ use $dx$

$R = 10$
$r = e^x$

$$V = \int_1^2 \pi \left( 10^2 - (e^x)^2 \right) dx$$

$$= \pi \int_1^2 \left( 100 - e^{2x} \right) dx$$

{ u-substitution $u = 2x$

$$= \pi \left[ 100x - \frac{1}{2} e^{2x} \Big|_1^2 \right]$$

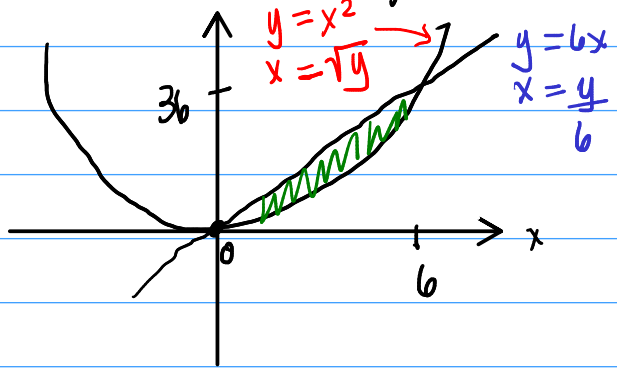
$$= \pi \left[ \left( 200 - \frac{1}{2} e^4 \right) - \left( 100 - \frac{1}{2} e^2 \right) \right]$$

$$\approx 240.003394$$

**Ex** Set up an integral for the volume of the solid obtained by rotating the region enclosed by

$y = x^2$ and $y = 6x$

about the specified axis. DO NOT evaluate.




intersection pts

$x^2 = 6x$

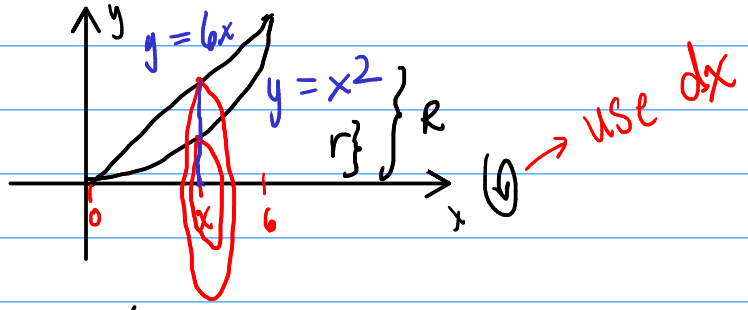
$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$ OR $6$

$y = 6(0) = 0$ $\quad$ $y = 6(6) = 36$

a) about x-axis




use dx

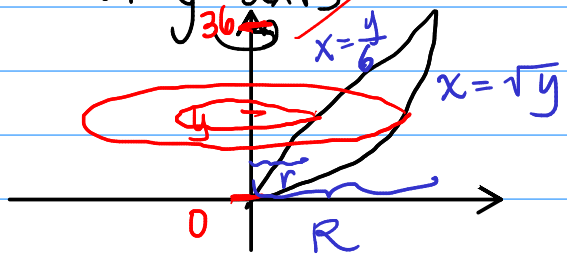
$R = 6x$

$r = x^2$

$$V = \int_0^6 \pi \left[ (6x)^2 - (x^2)^2 \right] dx$$

b) about y-axis

use dy




$R = \sqrt{y}$

$r = \frac{y}{6}$

$$V = \int_0^{36} \pi \left[ (\sqrt{y})^2 - \left(\frac{y}{6}\right)^2 \right] dy$$

NOTE: Quiz Friday: Improper Integrals & Area